A Mathematical Formulation and Tabu Search Approach for the Road-Rail Assignment Problem

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The Physical Internet Paradigm

Physical Internet (PI or π)

- Worldwide open logistics network.
- Transportation and handling of the freight is automated and synchronized.
- Products are handled in the same way as data packets over digital networks.

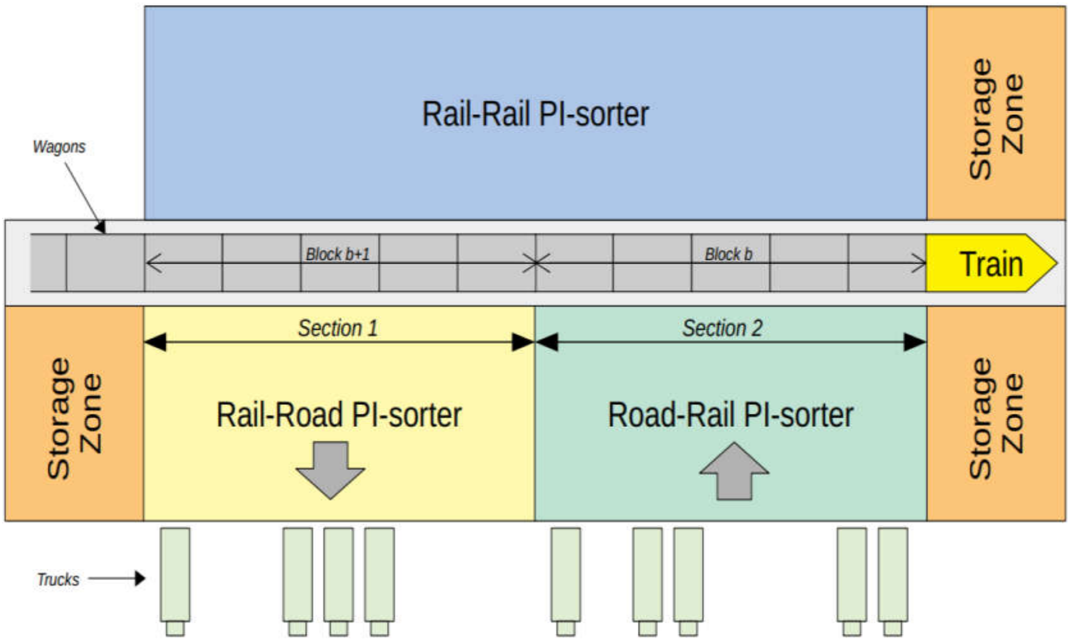
Physical Internet elements

- **PI-Containers**: Smart modular containers with standardized dimensions.
- **PI-Movers**: *For example:* PI-Conveyors, PI-Handlers, PI-Carriers...
- **PI-Nodes**: Locations such as: PI-Hubs, PI-Transits, PI-Sorters...

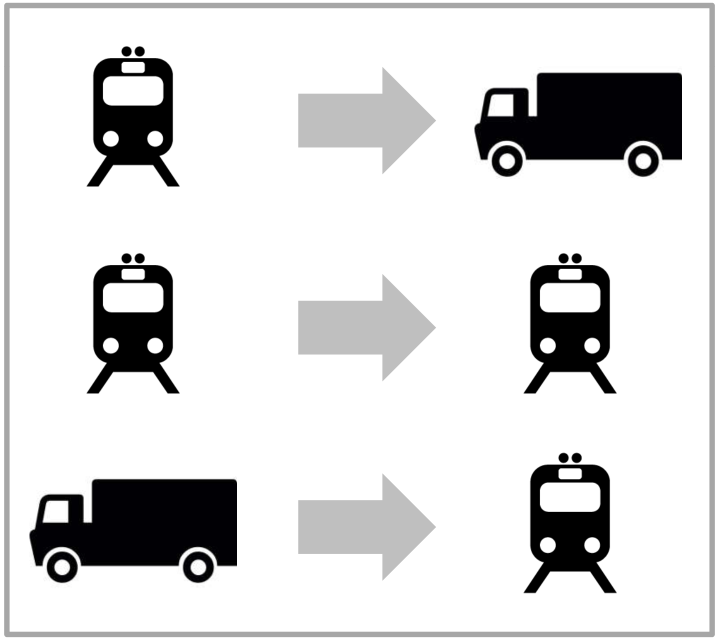
Road-Rail PI-hub

Road-Rail PI-hubs are designed to transfer PI-containers from trains to other trains, from trucks to trains and from trains to trucks.

Road-Rail PI-hubs layout



Flow of containers



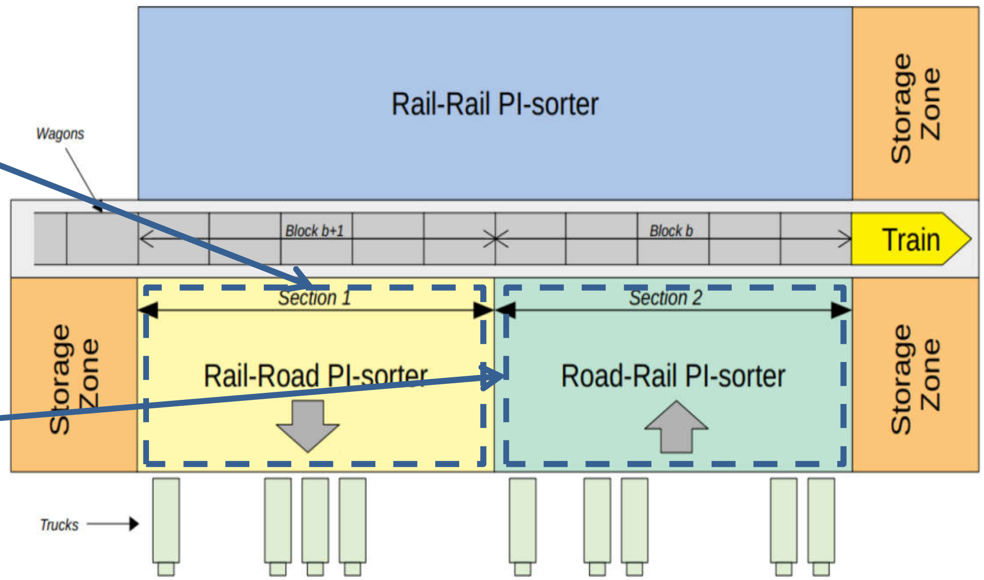
Review of the literature

PI-Hubs:

Problem	Author	Year
A rail-road PI-hub allocation problems: model and heuristic	Walha et al.	2014
Routing management in physical internet crossdocking hubs: Study of grouping strategies for truck loading	Pach et al.	2014
Functional Design of Physical Internet Facilities: A Road-Based Transit Center	Meller et al.	2012

Walha et al. (2014) formulated the Rail-Road allocation problem, where PI-containers are unloaded from wagons to outgoing trucks.

Our research focuses on the Road-Rail section which deals with the transfer of the PI-containers from trucks to wagons



Mathematical formulation

Input parameters (1/2)

Dimensions

N : total number of containers;

K : number of docks;

D : number of destinations;

W : number of wagons to load with PI-containers;

T : number of time periods;

H : number of trucks;

Q : wagon's capacity (useful length);

Indices

i : indices of the containers;

k : indices of the docks;

d : indices of the destinations;

w : indices of the wagons;

h : indices of the trucks;

Docks and Wagons Positions

P_k : position of the center of the dock k starting from the right axis of the Road-Rail PI-sorter zone;

R_w : position of the center of the wagon w starting from the right axis of the Road-Rail PI- sorter zone;

Input parameters (2/2)

Containers' data

L_i : length of container i;

$$A_{hi} = \begin{cases} 1, & \textit{if the container } i \textit{ is in the truck } h \\ 0, & \textit{Otherwise} \end{cases}$$

$$S_{di} = \begin{cases} 1, & \textit{if } d \textit{ is the destination of the container } i \\ 0, & \textit{Otherwise} \end{cases}$$

Other parameters

α : weighting factor for the number of used wagons;

β : weighting factor for the total distance traveled by containers;

M : A big positive number, $M \geq \max(W,\ 5 * 20m)$.

Mathematical formulation

Decision variables

Binary decision variables

$$x_{iw} = \begin{cases} 1, & \textit{if the container } i \textit{ is assigned to the wagon } w \\ 0, & \textit{Otherwise} \end{cases}$$

$$y_{hk} = \begin{cases} 1, & \textit{if the truck } h \textit{ is assigned to the dock } k \\ 0, & \textit{Otherwise} \end{cases}$$

$$u_w = \begin{cases} 1, & \textit{if the wagon } w \text{ is used} \\ 0, & \textit{Otherwise} \end{cases}$$

$$z_{iwk} = \begin{cases} 1, & \textit{if the container } i \textit{ is in a truck that is assigned to the dock } k, \\ & \textit{and the container } i \textit{ is assigned to the wagon } w \\ 0, & \textit{Otherwise} \end{cases}$$

$$e_{wd} = \begin{cases} 1, & \textit{if } d \textit{ is the destination of the wagon } w \\ 0, & \textit{Otherwise} \end{cases}$$

Continuous decision variables

d_{iw} : distance traveled by the container i to the wagon w;

Mathematical formulation

Objective function

The objective of the MILP model is to minimize the weighted sum of both the number of used wagons and the total internal traveled distance of PI-containers:

Minimize:

$$\alpha \sum_{w=1}^{W} u_w + \beta \sum_{i=1}^{N} \sum_{w=1}^{W} d_{iw}$$

Where **α** and **β** are the weighting factors for the number of used wagons and the total distance traveled by containers respectively.

Mathematical formulation

Constraints (1/4)

Assignment constraints (1/2)

$$\sum_{w=1}^{W} x_{iw} = 1 \quad (\forall\, i = 1 \dots N) \tag{2}$$

$$\sum_{i=1}^{N} x_{iw}\, L_i \leq Q \quad (\forall\, w = 1 \dots W) \tag{3}$$

$$x_{iw} + x_{jw} \leq \sum_{d=1}^{D} S_{di}\, S_{dj} + 1 \quad (\forall\, i, j = 1 \dots N, \forall\, w = 1 \dots W, i \neq j) \tag{4}$$

$$\sum_{h=1}^{H} y_{hk} \leq 1 \quad (\forall\, k = 1 \dots K) \tag{5}$$

$$\sum_{k=1}^{K} y_{hk} = 1 \quad (\forall\, h = 1 \dots H) \tag{6}$$

$$x_{iw} \leq u_w \quad (\forall\, i = 1 \dots N, \forall\, w = 1 \dots W) \tag{7}$$

Mathematical formulation

Constraints (2/4)

Assignment constraints (2/2)

$$e_{wd} \leq S_{di} + 1 - x_{iw} \quad (\forall\, i = 1 \ldots N, \forall\, w = 1 \ldots W, \forall\, d = 1 \ldots D) \tag{8}$$

$$u_w = \sum_{d=1}^{D} e_{wd} \quad (\forall\, w = 1 \ldots W) \tag{9}$$

$$|w_1 - w_2| + 1 \leq \sum_{w=1}^{W} e_{wd} + M\left(2 - \left(e_{w_1 d} + e_{w_2 d}\right)\right)$$
$$(\forall\, d = 1 \ldots D, \forall\, w_1, w_2 = 1 \ldots W, w_1 \neq w_2) \tag{10}$$

$$|w_1 - w_2| + 1 \leq \sum_{w=1}^{W} u_w + M\left(2 - \left(u_{w_1} + u_{w_2}\right)\right)$$
$$(\forall\, w_1, w_2 = 1 \ldots W, w_1 \neq w_2) \tag{11}$$

$$u_1 = 1 \tag{12}$$

Mathematical formulation

Constraints (3/4)

Distance constraints

$$d_{iw} \geq |P_k - R_w| - M(1 - z_{iwk}) \quad (\forall\, i = 1 \dots N, \forall\, w = 1 \dots W, \forall\, k = 1 \dots K, \forall\, h = 1 \dots H) \quad (13)$$

$$z_{iwk} A_{hi} \leq y_{hk} \quad (\forall\, i = 1 \dots N, \forall\, k = 1 \dots K, \forall\, h = 1 \dots H, \forall\, w = 1 \dots W) \quad (14)$$

$$\sum_{k=1}^{K} z_{iwk} = x_{iw} \quad (\forall\, i = 1 \dots N, \forall\, w = 1 \dots W) \quad (15)$$

$$x_{iw}, y_{hk}, u_w, z_{iwk}, e_{wd} \in \{0, 1\} \quad (\forall\, i = 1 \dots N, \forall\, w = 1 \dots W, \forall\, k = 1 \dots K, \forall\, h = 1 \dots H, \forall\, d = 1 \dots D) \quad (16)$$

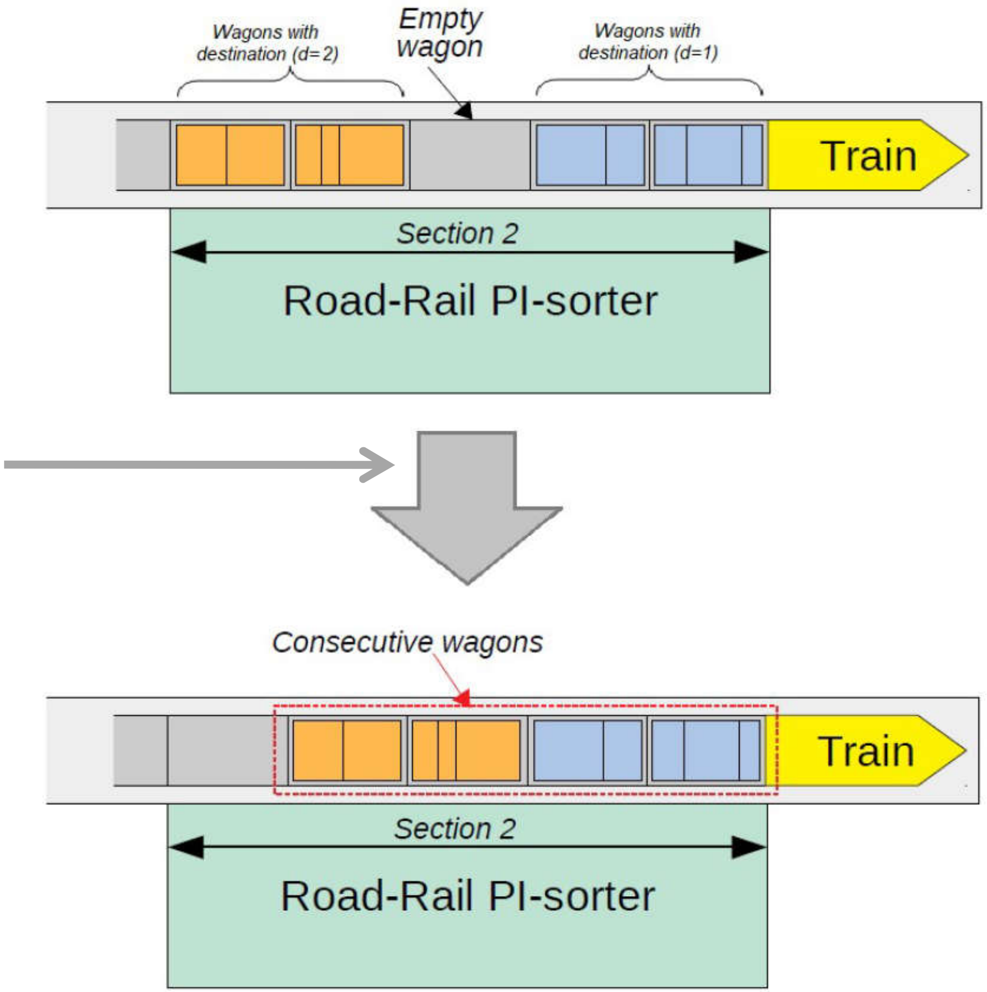
$$d_{iw} \geq 0 \quad (\forall\, i = 1 \dots N, \forall\, w = 1 \dots W) \quad (17)$$

Constraints (4/4)

All the used wagons must be consecutive

$$|w_1 - w_2| + 1 \leq \sum_{w=1}^{W} u_w + M\left(2 - \left(u_{w_1} + u_{w_2}\right)\right)$$

$$(\forall\, w_1, w_2 = 1 \ldots W, w_1 \neq w_2) \qquad (11)$$

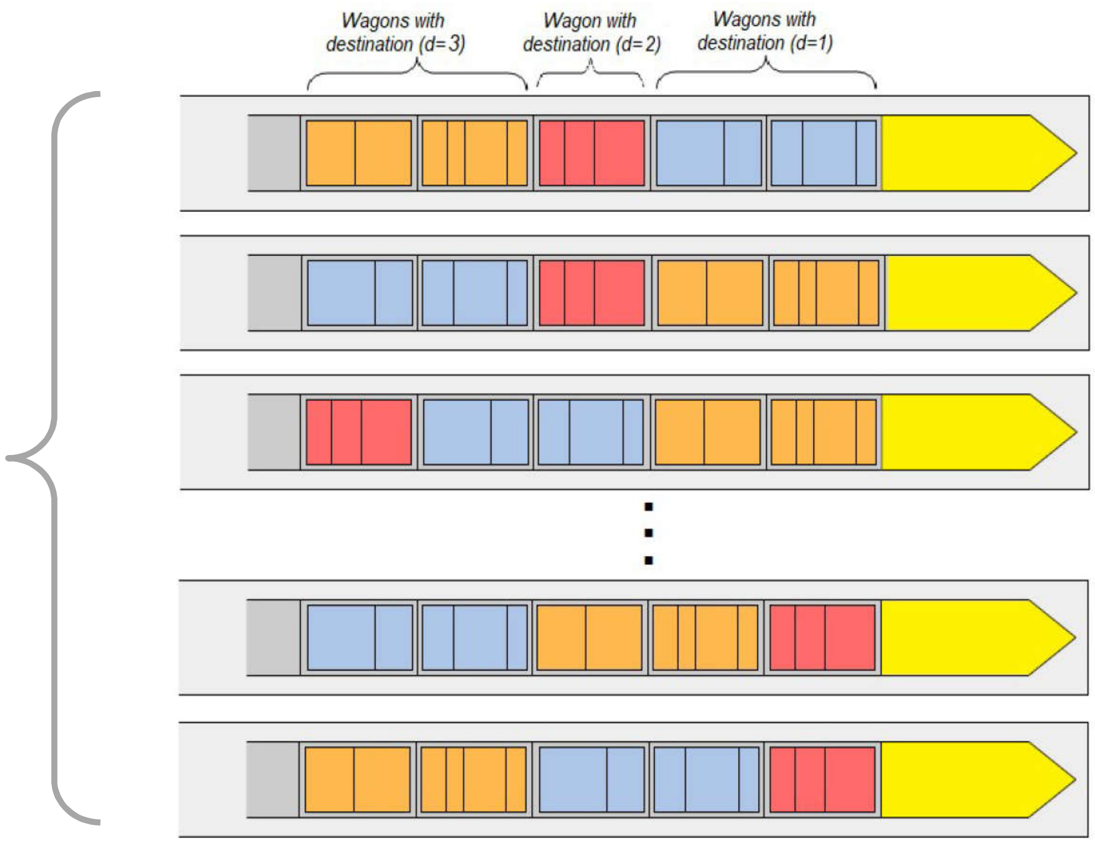


Proposed Tabu Search for the Road-Rail Problem

The proposed tabu search is composed of three steps:

- Improving the initial grouping of the first fit algorithm;
- Generating all the possible combinations of wagons of the initial grouping;
- Improving the trucks' assignment.

Example of grouping solutions generated in the second step of the tabu search



Proposed Tabu Search for the Road-Rail Problem

Solving Process

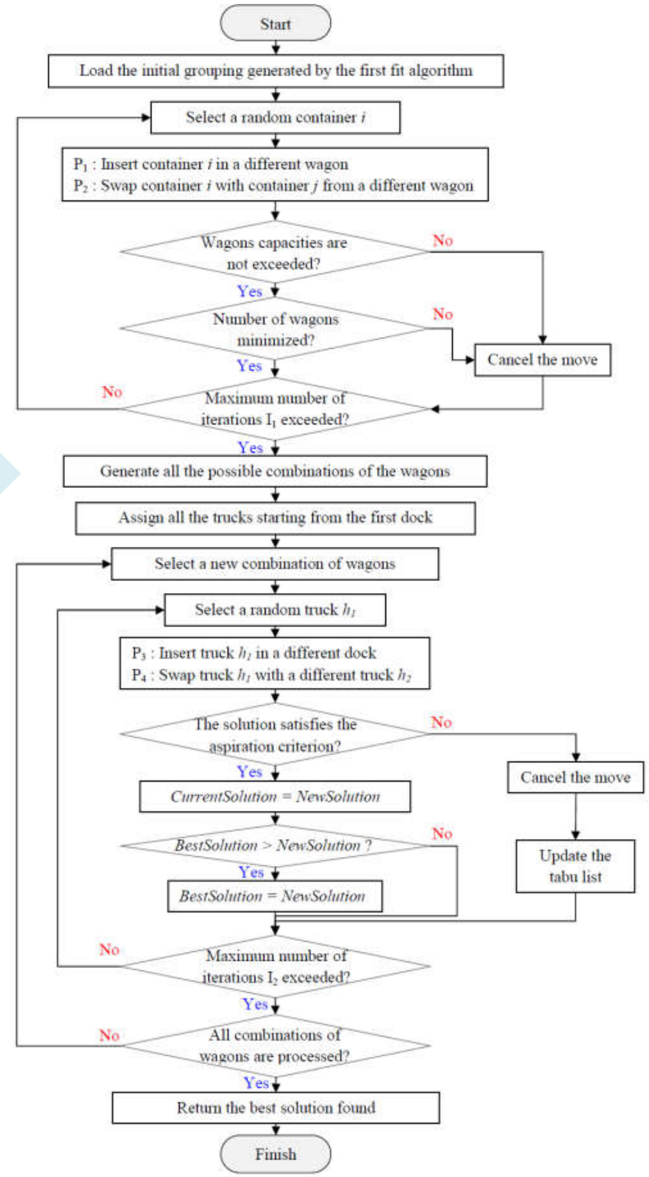
Each container is loaded in the first available wagon

Improve the initial grouping of the containers:
- two local search moves (Insert, swap) are performed with different probability (P1 + P2 = 1)

Generate all the possible combinations of the best grouping found in the first step

Improve the assignment of the trucks to the docks:
- two local search moves (Insert, swap) are performed with different probability (P1 + P2 = 1)

Return the best solution found



Results of CPLEX and Tabu search on instances

Comparing CPLEX and tabu search on small instances

			CPLEX (Optimal)			Tabu Search			
N	D	H	Used Wagons	Distance	CPU Time (s)	Used Wagons	Distance	CPU Time (s)	Distance Gap (%)
6	2	2	2	23.571	160.416	2	23.571	1.387	0.000%
		3	2	22.857	177.054	2	22.857	1.476	0.000%
	3	2	3	43.572	1220.394	3	43.572	2.707	0.000%
		3	3	42.143	819.489	3	42.143	3.650	0.000%
8	2	2	2	45.714	1094.823	2	46.427	2.177	1.560%
		3	2	64.285	3347.694	2	64.285	3.515	0.000%

Results obtained with the tabu search on large instances

N	D	H	Used Wagons	Distance	CPU Time (s)
10	2	5	4	28.572	4.566
		8	4	51.429	6.899
	3	5	4	62.858	4.822
		8	4	47.142	7.107
15	2	7	5	36.782	7.733
		10	5	23.214	5.356
	3	7	5	51.784	8.514
		10	5	70.356	9.451

Conclusion and Future Works

Conclusion

- A mixed integer linear programming MILP formulation of the Road-Rail assignment problem was proposed;
- The objective of the model was presented as a weighted sum of two objectives:

  - the number of used wagons
  - the total travel distance by PI-containers from the docks to wagons.

- A first fit based heuristic and a tabu search meta-heuristic were suggested to solve the proposed MILP model;
- The proposed tabu search was tested on several instances and gave good quality results.

Future Works

- Future works will be conducted on optimizing the formulation of the proposed mathematical model to reduce complexity to solve large instances;
- More tests will be conducted to test the robustness of the proposed methods on multiple instances.

Thanks for your time and attention !

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